

## 68. MATHEMATICS

### Differential and Integral Calculus (20 Marks)

**Partial Differentiation** : Introduction - Functions of two variables - Neighborhood of a point  $(a, b)$  - Continuity of a Function of two variables, Continuity at a point - Limit of a Function of two variables - Partial Derivatives - Geometrical representation of a Function of two Variables - Homogeneous Functions.

Theorem on Total Differentials - Composite Functions - Differentiation of Composite Functions - Implicit Functions - Equality of  $f_{xy}(a, b)$  and  $f_{yx}(a, b)$  - Taylor's theorem for a function of two Variables - Maxima and Minima of functions of two variables - Lagrange's Method of undetermined - multipliers.

**Curvature and Evolutes** : Introduction - Definition of Curvature - Radius of Curvature - Length of Arc as a Function, Derivative of arc - Radius of Curvature - Cartesian Equations - Newtonian Method - Centre of Curvature - Chord of Curvature.

**Evolutes**: Evolutes and Involutes - Properties of the evolute.

**Envelopes** : One Parameter Family of Curves - the family of straight lines - Definition - Determination of Envelope.

**Lengths of Plane Curves** : Introduction - Expression for the lengths of curves  $y = f(x)$  - Expressions for the length of arcs  $x = f(y)$ ;  $x = f(t)$ ,  $y = \phi(t)$ ;  $r = f(\theta)$

**Volumes and Surfaces of Revolution** : Introduction - Expression for the volume obtained by revolving about either axes - Expression for the volume obtained by revolving about any line - Area of the surface of the frustum of a cone - Expression for the surface of revolution - Pappus Theorems - Surface of revolution.

### Differential Equations (20 Marks)

**Differential Equations of first order and first degree**: Introduction - Equations in which Variables are Separable - Homogeneous Differential Equations - Differential Equations Reducible to Homogeneous Form - Linear Differential Equations - Differential Equations Reducible to Linear Form - Exact differential equations - Integrating Factors - Change in variables - Total Differential Equations - Simultaneous Total Differential Equations - Equations of the form  $\frac{dx}{P} = \frac{dy}{Q} = \frac{dz}{R}$

**Differential Equations first order but not of first degree**: Equations Solvable for  $p$  - Equations Solvable for  $y$  - Equations Solvable for  $x$  - Equations that do not contain  $x$  (or  $y$ )- Equations Homogeneous in  $x$  and  $y$  - Equations of the First Degree in  $x$  and  $y$  - Clairaut's equation. **Applications of First Order Differential Equations** : Growth and Decay - Dynamics of Tumour Growth - Radioactivity and Carbon Dating - Compound Interest - Orthogonal Trajectories.

**Higher order Linear Differential Equations**: Solution of homogeneous linear differential equations with constant coefficients - Solution of non-homogeneous differential equations  $P(D)y = Q(x)$  with constant coefficients by means of polynomial operators when  $Q(x) = e^{ax}$ ,  $b \sin ax/b \cos ax, bx^k$ ,  $V e^{ax}$  - Method of undetermined coefficients. Method of variation of parameters - Linear differential equations with non constant coefficients - The Cauchy - Euler Equation - Legendre's Linear Equations - Miscellaneous Differential Equations.

**Partial Differential Equations**: Formation and solution- Equations easily integrable - Linear equations of first order.

**Real Analysis (20 Marks)**

**Sequences:** Limits of Sequences- A Discussion about Proofs-Limit Theorems for Sequences- Monotone Sequences and Cauchy Sequences -Subsequences-Lim superiors and Lim inferior- Series-Alternating Series and Integral Test .

**Continuity:** Continuous Functions -Properties of Continuous Functions -Uniform Continuity - Limits of Functions

**Differentiation:** Basic Properties of the Derivative - Mean Value Theorems - L'Hospital Rule - Taylor's Theorem.

**Integration :** The Riemann Integral - Properties of Riemann Integral-Fundamental Theorem of Integral Calculus.

**Algebra (20 Marks)**

**Groups:** Definition and Examples of Groups- Elementary Properties of Groups-Finite Groups - Subgroups -Terminology and Notation -Subgroup Tests - Examples of Subgroups.

**Cyclic Groups:** Properties of Cyclic Groups - Classification of Subgroups - Cyclic Groups.

**Permutation Groups:** Definition and Notation -Cycle Notation-Properties of Permutations -A Check Digit Scheme Based on  $D_5$ . Isomorphisms ; Motivation- Definition and Examples - Cayley's Theorem Properties of Isomorphisms -Automorphisms-Cosets and Lagrange's Theorem Properties of Cosets - Lagrange's Theorem and Consequences-An Application of Cosets to Permutation Groups -The Rotation Group of a Cube and a Soccer Ball.

**Normal Subgroups and Factor Groups:** Normal Subgroups-Factor Groups -Applications of Factor Groups -Group Homomorphisms - Definition and Examples -Properties of Homomorphisms -The First Isomorphism Theorem.

**Introduction to Rings:** Motivation and Definition -Examples of Rings -Properties of Rings - Subrings.

**Integral Domains:** Definition and Examples - Fields -Characteristics of a Ring.

**Ideals and Factor Rings:** Ideals -Factor Rings -Prime Ideals and Maximal Ideals.

**Ring Homomorphisms:** Definition and Examples-Properties of Ring- Homomorphisms.

**Linear Algebra (20 Marks)**

**Vector Spaces:** Vector Spaces and Subspaces -Null Spaces, Column Spaces, and Linear Transformations

-Linearly Independent Sets; Bases -Coordinate Systems -The Dimension of a Vector Space-Rank-Change of Basis - Eigenvalues and Eigenvectors - The Characteristic Equation - Diagonalization -Eigenvectors and Linear Transformations -Complex Eigenvalues - Applications to Differential Equations.

**Orthogonality and Least Squares :** Inner Product, Length, and Orthogonality -Orthogonal Sets -Orthogonal Projections - The Gram-Schmidt orthogonalization Process.